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$$1. V = \frac{V_0}{\sqrt{r^2 - 1}}$$

$$V \sqrt{r^2 - 1} = V_0 \quad \checkmark$$

$$\sqrt{r^2 - 1} = \frac{V_0}{V} \quad \checkmark$$

(2) ✓

$$\textcircled{1} = \frac{V_0^2}{V^2}$$

$$\textcircled{2} = \frac{V_0^2 + 1}{V^2}$$

$$f = \frac{V_0^2 + 1}{V^2}$$

this was correct!

$$2. i) 5(3x-2) - 2(4x-3) + 25 = 0$$

$$15x - 10 - 8x + 6 + 25 = 0 \quad \checkmark$$

$$7x + 21 = 0 \quad \checkmark$$

$$7x = -21 \quad \checkmark$$

$$x = -3 \quad \checkmark$$

(4) ✓

$$ii) 6x^2 + 13x - 5 = 0$$

$$6x^2 + 10x + 3x - 5 = 0$$

$$2x(3x+5) + 1(3x-5) = 0$$

$$(2x-1)(3x+5)$$

$$6x - 5 = -30$$

$$10 - 3$$

$$\text{checking } 6x^2 + 10x$$

$$6x^2 + 15x - 2x - 5 = 0$$

$$15 - 2$$

$$3x(2x+5) - 1(2x+5) = 0 \quad \checkmark$$

$$(3x-1)(2x+5) \quad \checkmark$$

$$3x-1=0 \quad 2x+5=0 \quad \checkmark$$

$$3x=1 \quad 2x=-5$$

$$x = \frac{1}{3} \quad \checkmark \quad x = -\frac{5}{2} \quad \checkmark$$

(5) ✓

$$\text{iii) } 5 \times 10^{3x+2} = 215 \quad (\div 5)$$

$$10^{3x+2} = 43 \quad \checkmark$$

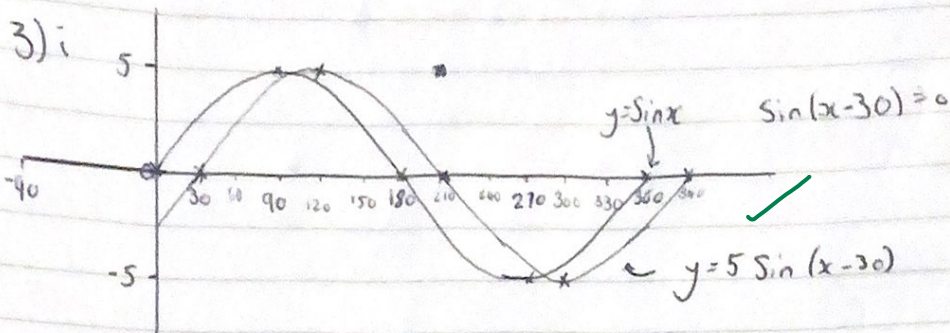
$$3x+2 = \log_{10}(43) \quad \checkmark$$

$$3x = \log_{10}(43) - 2 \quad \checkmark$$

$$x = \frac{\log_{10}(43) - 2}{3} \quad \checkmark$$

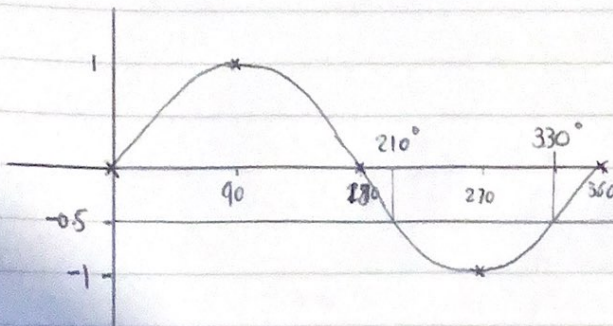
$$x = -0.12 \quad \checkmark$$

⑥ ✓



④ ✓

$$\text{ii} \quad \sin 2x = (-0.5)$$



$$\sin^{-1}(-0.5) = -30$$

$$-30 + 360 = 330^\circ \quad \checkmark$$

$$330^\circ - 270^\circ = 60^\circ$$

$$270^\circ + 60^\circ = 210^\circ \quad \checkmark$$

$$2x = 210^\circ, 330^\circ, 210^\circ + 360^\circ, 330^\circ + 360^\circ \quad \checkmark$$

$$2x = 210^\circ, 330^\circ, 570^\circ, 690^\circ \quad \checkmark (\div 2) \text{ for } x$$

$$x = 105^\circ, 165^\circ, 285^\circ, 345^\circ \quad \checkmark$$

⑨ ✓

4.  $z_1 = 3 - j5$   $z_2 = -2 + j4$

a)  $z_1 - z_2$

$$(3 - j5) - (-2 + j4) = 5 - j9 \quad \checkmark$$

② ✓

b)  $z_1 z_2$

$$\begin{aligned} (3 - j5)(-2 + j4) &= -6 + j12 + j10 - j^2 20 = -6 + j22 - (-1) \times 20 \quad \checkmark \\ &= -6 + 20 + j22 \quad \checkmark \\ &= 14 + j22 \quad \checkmark \end{aligned}$$

④ ✓

c)  $\frac{z_1}{z_2}$

$$\frac{(3 - j5)(-2 - j4)}{(-2 + j4)(-2 - j4)} = \frac{-6 - j12 + j10 + j^2 20}{4 + j8 - j^2 16} = \frac{-6 - j2 + (-1)20}{4 - (-1) \times 16}$$

$$= \frac{-6 - 20 - j2}{4 - 16} = \frac{-26 - j2}{-20} = \frac{26}{20} + \frac{j2}{20} = \frac{13}{10} + \frac{j1}{10}$$

⑦ ✓

5.  $A = \begin{pmatrix} 4 & -1 \\ 2 & -3 \end{pmatrix}$   $B = \begin{pmatrix} 5 & 0 \\ -2 & 6 \end{pmatrix}$

a)  $3A - 2B$

$$3 \begin{pmatrix} 4 & -1 \\ 2 & -3 \end{pmatrix} - 2 \begin{pmatrix} 5 & 0 \\ -2 & 6 \end{pmatrix} = \begin{pmatrix} 12 & -3 \\ 6 & -9 \end{pmatrix} - \begin{pmatrix} 10 & 0 \\ -4 & 12 \end{pmatrix} = \begin{pmatrix} 2 & -3 \\ 10 & -21 \end{pmatrix} \quad \checkmark$$

③ ✓

b)  $AB$

$$\begin{aligned} r_1 \begin{pmatrix} 4 & -1 \end{pmatrix} \times \begin{matrix} c_1 \\ c_2 \end{matrix} \begin{pmatrix} 5 & 0 \\ -2 & 6 \end{pmatrix} &= \begin{pmatrix} 4 \times 5 + (-1) \times (-2) & 4 \times 0 + (-1) \times 6 \\ 2 \times 5 + (-3) \times (-2) & 2 \times 0 + (-3) \times 6 \end{pmatrix} \\ &= \begin{pmatrix} 20 + 2 & 0 + (-6) \\ 10 + 6 & 0 + (-18) \end{pmatrix} = \begin{pmatrix} 22 & -6 \\ 16 & -18 \end{pmatrix} \quad \checkmark \end{aligned}$$

④ ✓

c)  $AB \neq BA$

$$\begin{aligned} r_1 \begin{pmatrix} 5 & 0 \end{pmatrix} \times \begin{matrix} c_1 \\ c_2 \end{matrix} \begin{pmatrix} 4 & -1 \\ 2 & -3 \end{pmatrix} &= \begin{pmatrix} 5 \times 4 + 0 \times 2 & 5 \times (-1) + 0 \times (-3) \\ -2 \times 4 + 6 \times 2 & -2 \times (-1) + 6 \times (-3) \end{pmatrix} = \begin{pmatrix} 20 + 0 & -5 + 0 \\ -8 + 12 & 2 + (-18) \end{pmatrix} \quad \checkmark \\ &= \begin{pmatrix} 20 & -5 \\ 4 & -16 \end{pmatrix} \quad \checkmark \end{aligned}$$

⑤ ✓  
JDP

d) Inverse of A

$$A = \begin{pmatrix} 4 & -1 \\ 2 & -3 \end{pmatrix}$$

$$(4 \times -3) - (-1 \times 2)$$
$$-12 - -2 = -10 \quad \checkmark$$

$$-\frac{1}{10} \begin{pmatrix} -3 & 1 \\ -2 & 4 \end{pmatrix} = \begin{pmatrix} \frac{3}{10} & -\frac{1}{10} \\ \frac{2}{10} & -\frac{4}{10} \end{pmatrix} \quad \checkmark$$

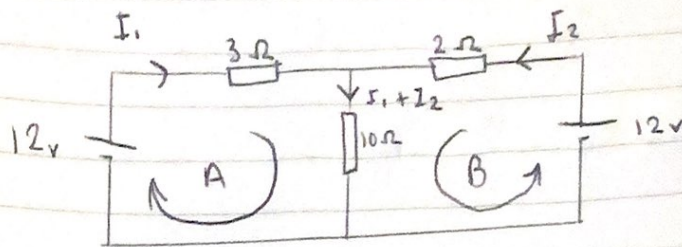
Checking

$$\begin{matrix} r_1 & r_2 \end{matrix} \begin{pmatrix} 4 & -1 \\ 2 & -3 \end{pmatrix} \times \begin{matrix} c_1 & c_2 \\ \begin{pmatrix} \frac{3}{10} & -\frac{1}{10} \\ \frac{2}{10} & -\frac{4}{10} \end{pmatrix} \end{matrix} = \begin{pmatrix} \frac{12}{10} + -\frac{2}{10} & -\frac{4}{10} + \frac{4}{10} \\ \frac{6}{10} + -\frac{6}{10} & -\frac{2}{10} + \frac{12}{10} \end{pmatrix} = \begin{pmatrix} \frac{10}{10} & 0 \\ 0 & \frac{10}{10} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \checkmark$$

$$\text{Inverse of } A = \begin{pmatrix} \frac{3}{10} & -\frac{1}{10} \\ \frac{2}{10} & -\frac{4}{10} \end{pmatrix} = \begin{pmatrix} \frac{3}{10} & -\frac{1}{10} \\ \frac{1}{5} & -\frac{2}{5} \end{pmatrix} \quad \checkmark$$

5

6)



$$\textcircled{A} \quad 12 = 3 \times I_1 + (I_1 + I_2) \times 10 \quad \checkmark$$

$$\textcircled{B} \quad 12 = 2 \times I_2 + (I_1 + I_2) \times 10 \quad \checkmark$$

$$\textcircled{A} \quad 12 = 3I_1 + 10I_1 + 10I_2 \quad \checkmark$$

$$\textcircled{B} \quad 12 = 2I_2 + 10I_1 + 10I_2 \quad \checkmark$$

$$\textcircled{A} \quad 13I_1 + 10I_2 = 12 \quad \checkmark$$

$$\textcircled{B} \quad 10I_1 + 12I_2 = 12 \quad \checkmark$$

$$I_1 = \begin{vmatrix} C & D \\ F & E \end{vmatrix} \quad I_2 = \begin{vmatrix} A & C \\ D & F \end{vmatrix}$$

$$\begin{vmatrix} A & B \\ D & E \end{vmatrix} \quad \begin{vmatrix} A & B \\ D & E \end{vmatrix}$$

$$I_1 = \frac{\begin{vmatrix} 12 & 10 \\ 12 & 12 \end{vmatrix}}{\begin{vmatrix} 13 & 10 \\ 10 & 12 \end{vmatrix}} = \frac{12 \times 12 - 10 \times 12}{13 \times 12 - 10 \times 10} = \frac{144 - 120}{156 - 100} = \frac{24}{56} \text{ A} = \frac{3}{7} \text{ A} \quad \checkmark$$

$$I_2 = \frac{\begin{vmatrix} 13 & 12 \\ 10 & 12 \end{vmatrix}}{\begin{vmatrix} 13 & 10 \\ 10 & 12 \end{vmatrix}} = \frac{13 \times 12 - 12 \times 10}{13 \times 12 - 10 \times 10} = \frac{156 - 120}{156 - 100} = \frac{36}{56} \text{ A} = \frac{9}{14} \text{ A} \quad \checkmark$$

$$I_1 \text{ resistor with } 3\Omega \text{ has } = \frac{3}{7} \text{ A} \quad \checkmark$$

$$I_1 + I_2 \text{ resistor with } 10\Omega = \frac{60}{56} \text{ A} = \frac{15}{14} \text{ A} \quad \checkmark$$

$$I_2 \text{ resistor with } 2\Omega \text{ has } = \frac{9}{14} \text{ A} \quad \checkmark$$

15 ✓

7.  $\begin{matrix} x & y \\ (-2, 3) & (4, 5) \end{matrix}$   $\frac{-2}{-6}$

i  $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 5}{-2 - 4} = \frac{-2}{-6} = -\frac{1}{3}$  X

$$y = -\frac{1}{3}x + c$$

$$\begin{matrix} x & y \\ (-2, 3) \end{matrix}$$

$$3 = -\frac{1}{3}(-2) + c$$

$$3 = \frac{2}{3} + c$$
 X

$$\frac{9}{3} = \frac{2}{3} + c \Rightarrow \frac{9}{3} - \frac{2}{3} = c$$

$$c = \frac{7}{3}$$
 X

$$y = -\frac{1}{3}x + \frac{8}{3}$$

ii)  $\begin{matrix} x & y \\ (2, 3) \end{matrix}$

$$m = \frac{3}{1} = 3$$
 ✓

correct for above error

$$y = 3x + c$$
 ✓

$$3 = 3 \times 2 + c$$
 ✓

$$3 = 6 + c$$
 ✓

$$3 - 6 = c$$

$$c = -3$$
 ✓

$$y = 3x - 3$$
 ✓

5 ✓

8.  $a = 1.50 \quad n = 50 \quad d = 0.70$

$$a + (n-1) \times d = 1.50 + (50-1) \times 0.70 = \pounds 35.80$$

✓ (4) ✓

9.

$$ar^{n-1}$$

$$10r^{5-1} = 700 \text{ rev/min} \quad \checkmark$$

$$r^4 = 70 \quad \checkmark$$

$$r = \sqrt[4]{70} \quad \checkmark$$

$$1\text{st speed} = 10 \times (\sqrt[4]{70})^{1-1} = 10 \text{ rev/min}$$

$$2\text{nd speed} = 10 \times (\sqrt[4]{70})^{2-1} = 29 \text{ rev/min} \quad \checkmark$$

$$3\text{rd speed} = 10 \times (\sqrt[4]{70})^{3-1} = 84 \text{ rev/min} \quad \checkmark$$

$$4\text{th speed} = 10 \times (\sqrt[4]{70})^{4-1} = 242 \text{ rev/min} \quad \checkmark$$

$$5\text{th speed} = 10 \times (\sqrt[4]{70})^{5-1} = 700 \text{ rev/min}$$

(8) ✓